ON STRESS ANALYSIS IN A HALF-PLANE WITH A CIRCULAR ORIFICE

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A treatment will be given of the problem of the elastic equilibrium of a half-plane with a circular orifice under the action of an arbitrary external loading on the circular and rectilinear parts of the boundary. The solution of the problem is given in a system of bipolar coordinates in the form of a stress function which is a generalization of the function obtained by Jeffery [1] for an eccentric ring.

1. The most general solution of the plane problem of elasticity for a region bounded by two circles, in particular for a half-plane with a circular orifice, was given by Jeffery [1]. This solution has been derived for the case when the boundaries of the region are subjected to arbitrary external forces represented by Fourier series and has been given in bipolar coordinates by means of a stress function which, by virtue of the required single-valuedness of the displacements, has a form [2]

$$g\Phi = G(\cosh\alpha - \cos\beta)\,\beta + B_0(\cosh\alpha - \cos\beta)\,\alpha + F(\beta\sin\beta + \nu\alpha\sinh\alpha) + H(\beta\sinh\alpha - \nu\alpha\sin\beta) + \sum_{n=1}^{\infty}[f_n^s(\alpha)\cos n\beta + f_n^s(\alpha)\sin n\beta] \qquad (1.1)$$

where (1.2)

$$f_n^c(\alpha) = A_n^c\cosh(n+1)\,\alpha + B_n^c\cosh(n-1)\,\alpha + C_n^c\sinh(n+1)\,\alpha + D_n^c\sinh(n-1)\,\alpha \qquad (n\geqslant 2)$$

$$f_n^s(\alpha) = A_n^s\cosh(n+1)\,\alpha + B_n^s\cosh(n-1)\,\alpha + C_n^s\sinh(n+1)\,\alpha + D_n^s\sinh(n-1)\,\alpha \qquad (n\geqslant 2)$$

$$f_1^c(\alpha) = A_1^c\cosh 2\alpha + B_1^c + C_1^c\sinh 2\alpha, \qquad f_1^s(\alpha) = A_1^s\cosh 2\alpha + C_1^s\sinh 2\alpha$$

However, in spite of a large number of particular problems solved by Jeffery's method, the question of the convergence of this solution in the general case remains open. It is not difficult to convince oneself that, for a half-plane with an orifice, solution (1.1) can lead to divergent series.

In fact, we will treat the simplest case of the equilibrium of a half-plane with an orifice, when the rectilinear boundary is free of stresses and the circular boundary is subjected to an arbitrary not self-equilibrating system of forces (equilibration takes place at infinity). Then the Jeffery's method for f_n^c and f_n^s on the rectilinear boundary (α = 0) leads to the following Expressions

$$nf_n^{c'}(0) = -nB_0 + nf_1^{c'}(0), \qquad nf_n^{s'}(0) = 2H \tag{1.3}$$

$$n(n^2-1)f_n^c(0) = -2nF, \qquad n(n^2-1)f_n^s(0) = -2G$$

where B_0, H, F and G can be determined in terms of the resultant of the forces acting on the circular boundary, and, in general, they are non-zero.

Thus, the necessary conditions for the convergence and two-fold differentiability (for the determination of the stresses) of series (1.1), namely

$$nf_n^{c'}(0) \to 0, \quad n(n^2-1)f_n^c(0) \to 0, \quad nf_n^{s'}(0) \to 0, \quad n(n^2-1)f_n^s(0) \to 0$$
$$\text{when } n \to \infty \tag{1.4}$$

are not fulfilled and, consequently, solution (1.1) is inapplicable in this case.

We mention without proof that, for the eccentric ring (not having infinitely remoted points), conditions (1.4) are fulfilled on the whole always when the external forces are in equilibrium.

Below, for the case of half-plane with an orifice, we give a solution of the plane problem of the theory of elasticity which is suitable for a wider range of problems than Jeffery's solution.

2. We will make use of a system of bipolar coordinates which can be obtained from Cartesian coordinates by means of the transformation [2]

$$x = a\sin\beta / (\cosh\alpha - \cos\beta), \qquad y = a\sinh\alpha / (\cosh\alpha - \cos\beta)$$

The region under consideration is bounded by the line α = 0 and the circle $\alpha = \gamma$.

Let the boundaries of the region be subjected to the external forces represented in the form of Fourier series

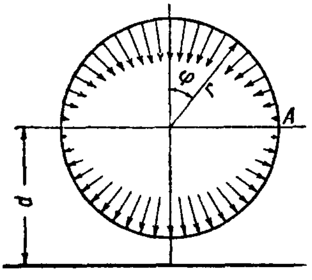

Fig. 1

$$a\tau_{\alpha\beta} = a_0' + \sum_{n=1}^{\infty}(a_n'\cos n\beta + b_n'\sin n\beta) \tag{2.1}$$
$$\text{for } \alpha = \gamma$$
$$a\sigma_\alpha = c_0' + \sum_{n=1}^{\infty}(c_n'\cos n\beta + d_n'\sin n\beta)$$

$$a\tau_{\alpha\beta} = a_0'' + \sum_{n=1}^{\infty}(a_n''\cos n\beta + b_n''\sin n\beta)$$
$$\text{for } \alpha = 0$$
$$a\sigma_\alpha = c_0'' + \sum_{n=1}^{\infty}(c_n''\cos n\beta + d_n''\sin n\beta)$$

Beside the conditions of representability by means of Fourier series, other necessary restrictions will become evident in the following.

The components of the resultant force and moment of the external forces applied to the boundary $\alpha = \gamma$ can be determined by well-known Formulas

$$X = -2\pi \sum_{n=1}^{\infty} n (a_n' - d_n') e^{-n\gamma}$$

$$Y = -2\pi \sum_{n=1}^{\infty} n (c_n' + b_n') e^{-n\gamma}$$

$$M = \frac{2\pi}{\sinh^2 \gamma} \sum_{n=0}^{\infty} a_n' e^{-n\gamma} \qquad (2.2)$$

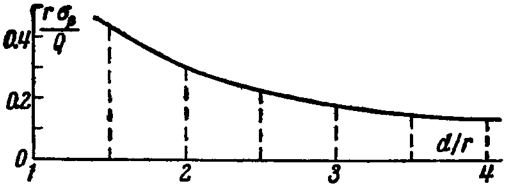

Fig. 2

The components of the resultant of the external forces applied to the boundary $\alpha = 0$ can also be unbounded.

Thus the problem leads to the determination of a biharmonic stress function, which in the considered region satisfies conditions of single-valuedness of the displacements and boundary conditions (2.1)

3. We will take the stress function in the form

$$g\Phi = G(\cosh\alpha - \cos\beta)\beta + B_0(\cosh\alpha - \cos\beta)\alpha + F(\beta\sin\beta + \nu\alpha\sinh\alpha) +$$

$$+ H(\beta\sinh\alpha - \nu\alpha\sin\beta) + (K\cosh\alpha - K\cos\beta + L\sinh\alpha + T\sin\beta)\ln(\cosh\alpha - \cos\beta) -$$

$$- L\alpha\sinh\alpha - T\alpha\sin\beta + (R\sin\beta + U\cosh\alpha - U\cos\beta + W\sinh\alpha)\tan^{-1}\frac{\sin\beta}{e^\alpha - \cos\beta} +$$

$$+ \frac{P\sinh\alpha\sin\beta}{\cosh\alpha - \cos\beta} + \sum_{n=1}^{\infty}[f_n^c(\alpha)\cos n\beta + f_n^s(\alpha)\sin n\beta] \qquad \left(\nu = \frac{\mu}{\lambda + 2\mu}\right) \qquad (3.1)$$

where $f_n^c(\alpha)$ and $f_n^s(\alpha)$ can be determined by Formulas (1.2); ν is a quantity depending on the elastic constants.

This function differs from Jeffery's stress function (1.1) by the presence of terms which are singular at the point $\alpha = \beta = 0$, at infinity, and which admit the possibility that the external forces at infinity are self-equilibrating. The non-singular terms $L\alpha \sinh \alpha$ and $T\alpha \sin \beta$ are necessary for the fulfillment of the condition of single-valuedness of the displacements. We consider the solution in more detail when the loading is symmetric.

In this case, the stress function will only contain terms which are even functions of β

$$g\Phi = B_0(\cosh\alpha - \cos\beta)\alpha + F(\beta\sin\beta + \nu\alpha\sinh\alpha) - L\alpha\sinh\alpha + (K\cosh\alpha - K\cos\beta +$$

$$+ L\sinh\alpha)\ln(\cosh\alpha - \cos\beta) + R\sin\beta\tan^{-1}\frac{\sin\beta}{e^\alpha - \cos\beta} + \sum_{n=1}^{\infty} f_n^c(\alpha)\cos n\beta \qquad (3.2)$$

The boundary conditions become

$$a\tau_{\alpha\beta} = \sum_{n=1}^{\infty} b_n'\sin n\beta, \quad a\sigma_\alpha = c_0' + \sum_{n=1}^{\infty} c_n'\cos n\beta \qquad \text{for } \alpha = \gamma \qquad (3.3)$$

$$a\tau_{\alpha\beta} = \sum_{n=1}^{\infty} b_n''\sin n\beta, \quad a\sigma_\alpha = c_0'' + \sum_{n=1}^{\infty} c_n''\cos n\beta \qquad \text{for } \alpha = 0$$

By determining the stresses on the boundaries of the region (3.2), developing them in Fourier series, and equating them to Expressions (3.3), we obtain four systems of Equations

$$
\begin{gathered}
-2f_2^{c'}(\gamma)+2f_1^{c'}(\gamma)\cosh\gamma-2B_0\cosh\gamma-2L\cosh\gamma+4L\sinh^2\gamma e^{-\gamma}-R\cosh\gamma+\\
+2R\sinh^2\gamma e^{-\gamma}-2K\sinh\gamma=2b_1'\\
-3f_3^{c'}(\gamma)+2\cdot 2f_2^{c'}(\gamma)\cosh\gamma-f_1^{c'}(\gamma)+B_0+4L\sinh^2\gamma e^{-2\gamma}+\frac{1}{2}R+2R\sinh^2\gamma e^{-2\gamma}=2b_2'\\
-(n+1)f_{n+1}^{c'}(\gamma)+2nf_n^{c'}(\gamma)\cosh\gamma-(n-1)f_{n-1}^{c'}(\gamma)+4L\sinh^2\gamma e^{-n\gamma}+\\
+2R\sinh^2\gamma e^{-n\gamma}=2b_n'\qquad(n\geqslant 3)
\end{gathered}\tag{3.4}
$$

$$
\begin{gathered}
2f_1^c(\gamma)-B_0\sinh 2\gamma-3F_0-2F\nu\sinh^2\gamma+4L\sinh^2\gamma-4L\sinh\gamma\cosh\gamma+\frac{3}{2}R-\\
-R\sinh\gamma\cosh\gamma+R\sinh^2\gamma-K\cosh 2\gamma=2c_0'\\
2\cdot 3f_2^c(\gamma)-2f_1^{c'}(\gamma)\sinh\gamma+2B_0\sinh\gamma+4F\cosh\gamma+4L\sinh^2\gamma e^{-\gamma}-2L\sinh\gamma-\\
-R\cosh\gamma-\mathrm{R}e^{-\gamma}+2R\sinh^2\gamma e^{-\gamma}+2K\cosh\gamma=2c_1'\\
3\cdot 4f_3^c(\gamma)-2\cdot 3f_2^c(\gamma)\cosh\gamma-2f_2^{c'}(\gamma)\sinh\gamma-F+4L\sinh^2\gamma e^{-2\gamma}+\frac{1}{2}R+\\
+2R\sinh^2\gamma e^{-2\gamma}-K=2c_2'\\
(n+1)(n+2)f_{n+1}^c(\gamma)-2(n^2-1)f_n^c(\gamma)\cosh\gamma+(n-1)(n-2)f_{n-1}^c(\gamma)-\\
-2f_n^{c'}(\gamma)\sinh\gamma+4L\sinh^2\gamma e^{-n\gamma}+2R\sinh^2\gamma e^{-n\gamma}=2c_n'\qquad(n\geqslant 3)
\end{gathered}\tag{3.5}
$$

$$
\begin{gathered}
-2f_2^{c'}(0)+2f_1^{c'}(0)-2B_0-R-2L=2b_1''\\
-3f_3^{c'}(0)+2\cdot 2f_2^{c'}(0)-f_1^{c'}(0)+B_0+\frac{1}{2}R=2b_2''\\
-(n+1)f_{n+1}^{c'}(0)+2nf_n^{c'}(0)-(n-1)f_{n-1}^{c'}(0)=2b_n''\qquad(n\geqslant 3)
\end{gathered}\tag{3.6}
$$

$$
\begin{gathered}
2f_1^c(0)-3F+\frac{3}{2}R-K=2c_0''\\
2\cdot 3f_2^c(0)+4F-2R+2K=2c_1''\\
3\cdot 4f_3^c(0)-2\cdot 3f_2^c(0)-F+{}^1/_2R-K=2c_2''\\
(n+1)(n+2)f_{n+1}^c(0)-2(n^2-1)f_n^c(0)+(n-1)(n-2)f_{n-1}^c(0)=2c_n''\quad(n\geqslant 3)
\end{gathered}\tag{3.7}
$$

These systems can easily be solved by means of the method set forth in [1]. Resulting solutions are

$$
\begin{gathered}
nf_n^{c'}(\gamma)=\frac{1}{\sinh\gamma}\left[f_1^{c'}(\gamma)-B_0-2Le^{-2\gamma}-\frac{R}{2}e^{-2\gamma}-2K\sinh\gamma e^{-\gamma}-2\sum_{p=1}^{n-1}b_p'e^{-p\gamma}\right]e^{n\gamma}+\\
+\frac{1}{\sinh\gamma}\left[-f_1^{c'}(\gamma)+B_0+2Le^{-2\gamma}-2L\sinh\gamma\cosh\gamma-2Ln\sinh^2\gamma+\frac{R}{2}e^{-2\gamma}-\right.\\
\left.-Rn\sinh^2\gamma+2K\sinh\gamma e^{\gamma}+2\sum_{p=1}^{n-1}b_p'e^{p\gamma}\right]e^{-n\gamma}\qquad(n\geqslant 2)
\end{gathered}\tag{3.8}
$$

$$
nf_n^{c'}(0)=\left[f_1^{c'}(0)-B_0-2L-\frac{R}{2}-2\sum_{p=1}^{n-1}b_p''\right]n+2L+2\sum_{p=1}^{n-1}pb_p''\qquad(n\geqslant 2)
$$

$$n\,(n^2-1)\,f_n^{\,c}(\gamma)=\frac{1}{2\sinh\gamma}\Big[f_1^{\,c\prime}(\gamma)-2Le^{-2\gamma}-\frac{R}{2}e^{-2\gamma}-2K\sinh\gamma e^{-\gamma}-B_0-$$

$$-2\sum_{p=1}^{n-1}b_p{}'e^{-p\gamma}\Big]ne^{n\gamma}+\frac{\cosh\gamma}{2\sinh^2\gamma}\Big[-f_1^{\,c\prime}(\gamma)+2Le^{-2\gamma}+\frac{R}{2}e^{-2\gamma}+$$

$$+2K\sinh\gamma e^{-\gamma}+B_0+2\sum_{p=1}^{n-1}b_p{}'e^{-p\gamma}\Big]e^{n\gamma}+\frac{1}{\sinh\gamma}\Big[-F+\sum_{p=1}^{n-1}p\,(b_p{}'+c_p{}')\,e^{-p\gamma}\Big]e^{n\gamma}+$$

$$+\sinh\gamma\,[2L+R]\,n^2e^{-n\gamma}+\frac{1}{2\sinh\gamma}\Big[f_1^{\,c\prime}(\gamma)-2Le^{-2\gamma}-Re^{-2\gamma}+\frac{R}{2}e^{2\gamma}-2K\sinh\gamma e^{\gamma}-$$

$$-B_0-2\sum_{p=1}^{n-1}b_p{}'e^{p\gamma}\Big]ne^{-n\gamma}+\frac{\cosh\gamma}{2\sinh^2\gamma}\Big[f_1^{\,c\prime}(\gamma)-2Le^{-2\gamma}-\frac{R}{2}e^{-2\gamma}-2K\sinh\gamma e^{\gamma}-B_0-$$

$$-2\sum_{p=1}^{n-1}b_p{}'e^{p\gamma}\Big]e^{-n\gamma}+\frac{1}{\sinh\gamma}\Big[F-2L\sinh^2\gamma+\sum_{p=1}^{n-1}p\,(b_p{}'-c_p{}')\,e^{p\gamma}\Big]e^{-n\gamma}\qquad(n\geqslant 2)$$

$$n\,(n^2-1)\,f_n^{\,c}(0)=-2nF+nR-2K+2\sum_{p=1}^{n-1}(n-p)\,pc_p{}''\qquad(n\geqslant 2)$$

$$f_1^{\,c}(\gamma)=c_0{}'+B_0\sinh\gamma\cosh\gamma+\frac{3}{2}F+F\,\nu\sinh^2\gamma+2L\sinh\gamma e^{-\gamma}-$$

$$-\frac{3}{4}R+\frac{R}{2}\sinh\gamma e^{-\gamma}+\frac{1}{2}K\cosh 2\gamma$$

$$f_1^{\,c}(0)=c_0{}''+\frac{3}{2}F-\frac{3}{4}R+\frac{1}{2}K$$

We will consider Expression for $nf_n^{\,c\prime}(\gamma)$ in detail. It can be rewritten in the form

$$nf_n^{c\prime}(\gamma)=\frac{1}{\sinh\gamma}\Big[f_1^{c\prime}(\gamma)-B_0-2Le^{-2\gamma}-\frac{R}{2}e^{-2\gamma}-2K\sinh\gamma e^{-\gamma}-2\sum_{p=1}^{\infty}b_p{}'e^{-p\gamma}\Big]e^{n\gamma}+$$

$$+\frac{1}{\sinh\gamma}\Big\{\Big[-f_1^{c\prime}(\gamma)+B_0+2Le^{-2\gamma}-2L\sinh\gamma\cosh\gamma-2Ln\sinh^2\gamma+\frac{R}{2}e^{-2\gamma}-$$

$$-Rn\sinh^2\gamma+2K\sinh\gamma e^{\gamma}+2\sum_{p=1}^{n-1}b_p{}'e^{p\gamma}\Big]e^{-n\gamma}+2\sum_{p=n}^{\infty}b_p{}'e^{(n-p)\gamma}\Big\}\qquad(n\geqslant 2)$$

The necessary condition that this Expression tends to zero means that the coefficient of $e^{n\gamma}$ must vanish

$$f_1^{c\prime}(\gamma)-B_0-2Le^{-2\gamma}-\frac{1}{2}Re^{-2\gamma}-2K\sinh\gamma e^{-\gamma}-2\sum_{p=1}^{\infty}b_p{}'e^{-p\gamma}=0$$

Analogically, from the requirements $nf_n^{\,c\prime}(0)\to 0$ and $n(n^2-1)f_n^{\,c}\to 0$ follows the necessity of satisfying following conditions

$$f_1^{c'}(\gamma) - B_0 - 2Le^{-2\gamma} - \frac{1}{2}\operatorname{Re}^{-2\gamma} - 2K\sinh\gamma e^{-\gamma} - 2\sum_{p=1}^{\infty} b_p' e^{-p\gamma} = 0$$

$$f_1^{c'}(0) - B_0 - 2L - \frac{1}{2}R - 2\sum_{p=1}^{\infty} b_p'' = 0$$

$$L + \sum_{p=1}^{\infty} pb_p'' = 0, \qquad F - \sum_{p=1}^{\infty} p(b_p' + c_p')\, e^{-p\gamma} = 0 \tag{3.9}$$

$$2F - R - 2\sum_{p=1}^{\infty} pc_p'' = 0, \qquad K - \sum_{p=1}^{\infty} p^2 c_p'' = 0$$

Last four conditions in (3.9) determine the constants L, R, F and K.

First two Equations in (3.9) together with last Equations in (3.8) make it possible to determine B_0 and the coefficients A_1^c, B_1^c and C_1^c for $f_1^c(\alpha)$ in Expression (1.2) . Remaining Equations (3.8) are sufficient for the determination of the constants A_n^c, B_n^c, C_n^c and D_n^c.

The problem with asymmetric loading of the region boundaries can be solved in complete analogy. In this case, for the constant coefficients of the stress function we obtain

$$G = -\frac{1}{\sinh^2\gamma}\sum_{p=0}^{\infty} a_p' e^{-p\gamma} + \coth\gamma \sum_{p=1}^{\infty} p\,(a_p' - d_p')\, e^{-p\gamma}$$

$$H = -\sum_{p=1}^{\infty} p\,(a_p' - d_p')\, e^{-p\gamma}$$

$$U = -\frac{2}{\sinh^2\gamma}\sum_{p=0}^{\infty} a_p' e^{-p\gamma} + 2\coth\gamma\sum_{p=1}^{\infty} p\,(a_p' - d_p')\, e^{-p\gamma} + 2\sum_{p=1}^{\infty} p^2 d_p'' \tag{3.10}$$

$$W = -2\sum_{p=1}^{\infty} p\,(a_p' - d_p')\, e^{-p\gamma} - 2\sum_{p=1}^{\infty} pa_p'', \quad P = \sum_{p=0}^{\infty} a_p'', \quad T = -\sum_{p=1}^{\infty} pd_p''$$

Expressions (3.9) and (3.10) make it possible to find the restrictions imposed on the external loading by the convergence condition; the Fourier coefficients of the loading on the rectilinear boundary must satisfy the requirement that the following series converge

$$\sum pa_p'', \qquad \sum pb_p'', \qquad \sum p^2 c_p'', \qquad \sum p^2 d_p''$$

There are no additional restrictions on the circular boundary.

Thus, all coefficients of the stress function (3.2) can be determined. It should be noted that the fulfillment of the neccessary conditions (3.9) will not be sufficient for convergence of the obtained series. As sufficient conditions it can be shown the convergence condition of the series

$$\sum nf_n^{c'}(\gamma), \qquad \sum nf_n^{c'}(0); \qquad \sum n(n^2-1)\, f_n^c(\gamma), \qquad \sum n(n^2-1)\, f_n^c(0)$$

however, it would appear that one could find weaker sufficient conditions.

4. As an example, we treat the half-plane with a circular orifice (Fig.1) under the action of loading distributed along the contour of the orifice according to the law

$$a\sigma_\alpha = p\cos\varphi = \frac{Q\sinh\gamma}{\pi}\frac{\cosh\gamma\cos\beta - 1}{\cosh\gamma - \cos\beta}, \qquad a\tau_{\alpha\beta} = 0$$

The rectilinear boundary is free from loading. Then

$$a_n' = b_n' = d_n' = a_n'' = b_n'' = c_n'' = d_n'' = 0, \qquad c_0' = \frac{1}{\pi}\int_0^\pi a\sigma_\alpha d\beta = -\frac{Q\sinh\gamma}{\pi}e^{-\gamma}$$

$$c_n' = \frac{2}{\pi}\int_0^\pi a\sigma_\alpha \cos n\beta\, d\beta = \frac{2Q\sinh^2\gamma}{\pi}e^{-n\gamma} \qquad (n\geqslant 1)$$

By formulas (3.9) we obtain

$$L = K = 0, \qquad F = \frac{Q}{2\pi}, \qquad R = \frac{Q}{\pi}$$

First two Equations (3.9), together with Equations (3.8), give with regard to (1.2)

$$A_1^c\cosh 2\gamma + B_1^c + C_1^c\sinh 2\gamma - B_0\sinh\gamma\cosh\gamma = \frac{Q}{2\pi}(\nu\sinh^2\gamma - \sinh\gamma e^{-\gamma})$$

$$2A_1^c\sinh 2\gamma + 2C_1^c\cosh 2\gamma - B_0 = \frac{Q}{2\pi}e^{-2\gamma}, \qquad A_1^c + B_1^c = 0, \qquad 2C_1^c - B_0 = \frac{Q}{2\pi}$$

$$A_n^c\cosh(n+1)\gamma + B_n^c\cosh(n-1)\gamma + C_n^c\sinh(n+1)\gamma + D_n^c\sinh(n-1)\gamma = 0$$

$$(n+1)A_n^c\sinh(n+1)\gamma + (n-1)B_n^c\sinh(n-1)\gamma + (n+1)C_n^c\cosh(n+1)\gamma +$$
$$+ (n-1)D_n^c\cosh(n-1)\gamma = -\frac{Q}{\pi}\sinh\gamma e^{-n\gamma}$$

$$A_n^c + B_n^c = 0, \qquad (n+1)C_n^c + (n-1)D_n^c = 0$$

Solving these Equations and substituting the found values into (3.2), we obtain final Expression for the stress function

$$g\Phi = \frac{Q}{2\pi}(2\coth^2\gamma - 2\coth\gamma - \nu\coth\gamma)(\alpha\cosh\alpha - \alpha\cos\beta) + \frac{Q}{2\pi}(\beta\sin\beta + \nu\alpha\sinh\alpha) +$$
$$+ \frac{Q}{\pi}\sin\beta\tan^{-1}\frac{\sin\beta}{e^\alpha - \cos\beta} + \frac{Q}{4\pi}(1 + \nu - 2\coth\gamma)\cosh 2\alpha\cos\beta +$$
$$+ \frac{Q}{4\pi}(-1 - \nu + 2\coth\gamma)\cos\beta + \frac{Q}{4\pi}(2\coth^2\gamma + 1 - 2\coth\gamma - \nu\coth\gamma)\sinh 2\alpha\cos\beta +$$
$$+ \frac{Q\sinh\gamma}{\pi}\sum_{n=2}^{\infty}\frac{\sinh n\gamma\sinh n\alpha\sinh(\gamma-\alpha) - n\sinh\gamma\sinh\alpha\sinh n(\gamma-\alpha)}{\sinh^2 n\gamma - n^2\sinh^2\gamma}e^{-n\gamma}\cos n\beta$$

Fig.2 shows the nature of the dependence of the stress σ_β at point A (Fig.1) on the position of the orifice relative to the rectilinear boundary of the half-plane.

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